

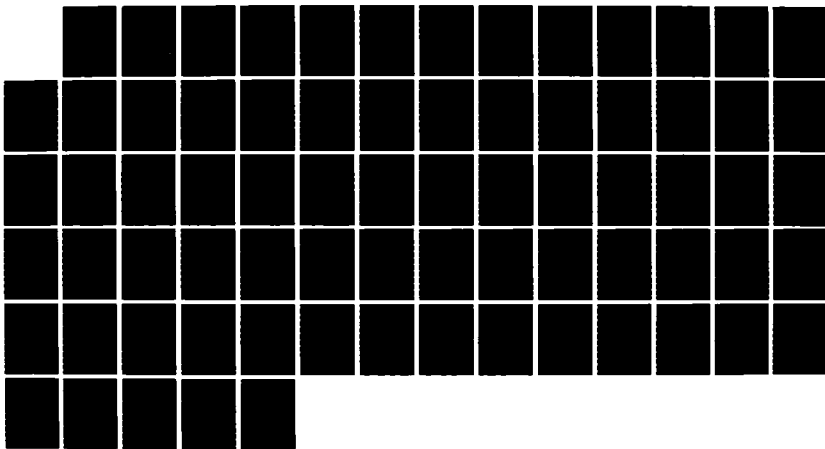
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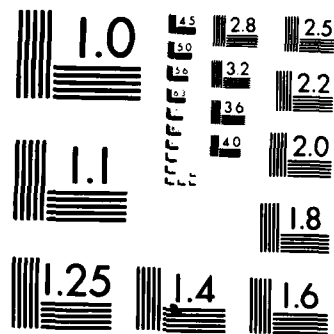
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STATISTICAL MODELS FOR PREDICTING
THE CHANGE IN MEAN MOTION OF A
SATELLITE OVER TIME INCLUDING
THE EFFECTS OF SOLAR FLUX

THESIS

James M. Burns, B.S.
Captain, USAF
AFIT/GSO/ENS/85D-5

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STATISTICAL MODELS FOR PREDICTING THE CHANGE IN
MEAN MOTION OF A SATELLITE OVER TIME
INCLUDING THE EFFECTS OF SOLAR FLUX

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Space Operations

James M. Burns, B.S.

Captain, USAF

December, 1985

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PREFACE

The purpose of this study was to fill a need of the Space Operations Directorate of the North American Aerospace Defense Command (NORAD). One of the missions of the Space Operations Directorate is to maintain positional data on all man made objects in space. Ocassionally some of these objects cannot be located. To find a lost object, it is neccessary to estimate where the object could be from historical positional data on the satellite. This is simple to do over a short time but the error increases rapidly over longer times because of changes in the satellite's period due to orbital perturbations. For near earth satellites one of the most important perturbations is atmospheric drag which is influenced by solar activity. A method to predict this change in period (or mean motion) from initial data was needed. Required characteristics for accomplishing this are simplicity and computational efficiency since many objects must be updated at any time.

The method developed in this thesis is simple and rapid. Tests were performed on actual data to verify the model.

I would like to thank all those who helped in the production of this thesis. First of all, I would like to thank my advisor, LtC Charles Ebeling, for his assistance and patience on this project. I would also like to thank Lt Terry Sparks of NORAD for his help in collecting the data needed to carry out this thesis. I have special thanks for Mary Browning and Pam McCarthy of the library for their help in locating some very obscure references.

James M. Burns

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NOTATION

n	Mean motion
\dot{n}	First time derivative of mean motion
\ddot{n}	Second time derivative of mean motion
$\dot{N}/2$	Difference between Brouwer and Kozai mean motion
P	Period of the orbit
\dot{P}	Change in period per day
t	Time
dt	Change in time
SF	Solar flux F10.7 value
S	Change in SF at time t
F	Change in SF at time $t-dt$
\bar{D}	Drag force
C	Drag coefficient
P	Atmospheric density
A	Satellite's cross sectional area
v	Relative satellite velocity
$\ddot{\bar{r}}$	Acceleration due to drag
$\dot{\bar{r}}$	Total satellite velocity
m	Satellite mass

ABSTRACT

This investigation derived a simple model to determine the change in mean motion over time when the actual values are unknown. A method was developed to include effects of solar flux by calculating an average value of n over 30 days. The model requires a knowledge of the mean motion for about 30 days before the time of interest to calculate this average.

The analysis was done using BMDP on a CDC Cyber 6000 computer using element set data from actual satellites.

This model does not attempt absolute accuracy, but is intended to be a method to quickly approximate a new mean motion when real values are not available. A limitation of this model is the amount of historical data and analyst judgement which are required.

STATISTICAL MODELS FOR PREDICTING THE CHANGE IN
MEAN MOTION OF A SATELLITE OVER TIME
INCLUDING THE EFFECTS OF SOLAR FLUX

I. Introduction

BACKGROUND

Satellites in near earth orbit (those at an altitude of under 1000 km) show a loss of orbital period known as decay. This decay is caused by the drag on a satellite due to the upper atmosphere. It would be a simple matter to account for this decay if the drag were constant. Drag, however, is not constant. It is a function of atmospheric density. This density is, in turn, a function of altitude and solar flux (changes in solar flux cause changes in the atmospheric density at all altitudes). It can be concluded, therefore, that the decay of a satellite is a function of its altitude and of the solar flux.

There are several ways to calculate and predict this decay. One method requires an accurate model of the upper atmosphere. The model must include both solar flux and altitude dependence. One such atmospheric model is the Jacchia model. Using this model and the known parameters of the satellite, it is possible, through astrodynamics, to calculate a very accurate near term orbit for the satellite. This method is very time consuming and requires up-to-the-minute knowledge of the satellite's position. It also decreases in accuracy as the position is predicted further into the future (1).

A second method is to calculate the rate of change in mean motion

(the number of orbital revolutions per day, known as n) of a satellite. This is called NDOT. If NDOT is known, then the mean motion of the satellite and its position (or "element set") can be predicted for any time. This prediction is known as "propagation of the element set." NDOT is found by a least squares fit of actual data to two different astrodynamic models (known as the SGP model and the GP4 model). This produces a fairly accurate model of the satellite's orbit in far less time than a complete model such as the Jacchia requires. It is also less accurate than the Jacchia model for the same time, but it loses accuracy less quickly than the Jacchia. Over longer times the new n produced from the simple model is more accurate than that from an exact model. That is, the model is valid for several days instead of hours. Both of the models are in use at the North American Aerospace Defense Command (NORAD).

While other models have been tried, none of them have outperformed these two. In tests done by NORAD, the models which provide some improvement in accuracy do so at the expense of greatly increased computer run times (2-3).

RESEARCH OBJECTIVE

A method that is accurate over long delta times and requires no knowledge of the satellite's current position is needed for predicting n . It should not require an increase in computer run times or size, above that of the current methods. It is not a replacement for any of the methods currently in use. It is to be considered as an additional method for an area where existing methods are weakest.

Research questions include:

- 1) Is there a statistical relation between n and solar flux?
- 2) What is the best statistical relation between n and \dot{n} ?
- 3) What is the best statistical relation between n and \ddot{n} ?
- 4) Is there a statistical relation between n and other orbital elements?

SCOPE

There are several types of possible errors in satellites orbits that must be allowed for. These errors may be divided into three main types. They are altitude errors, plane errors, and in-track errors. Altitude errors are errors in the satellites orbital altitude. Plane errors are errors in the satellites orbital plane and can include errors in inclination and errors in right ascension of node. In-track errors are time bias errors between the time a satellite should be at some place and the time it actually arrives there. Altitude errors are normally measured in units of length. Plane errors are measured in units of angle. In-track errors are measured in terms of time. These errors may be combined for a satellite and given in terms of absolute distance error, but that is not normally done in routine cases. This thesis will concentrate on in-track errors only.

This thesis will develop and test some methods of predicting mean motion of an orbit. Chapter II will begin this development by providing some background and basic theory of orbital decay. It will define the terms that will be used in this thesis, and will explore some of the current models and methods used for satellite decay. Chapter III will then develop the models that will be tested in this thesis. Chapter IV will

study the test results of these models and compare them to each other and a current model. Chapter V will then present the conclusions of this study. Extensive use will be made of tables and figures to illustrate the study that was done.

II. THEORY

ORBITAL PARAMETERS

A satellite's orbit is described by several parameters. There are several different sets of parameters used for this. These sets include the set of position and velocity vectors, the Keplerian element set, and the F&G series (3). The parameters used in this thesis are those used by NORAD, which are almost the same as those in the Keplerian element set. There are some added parameters which will be included in the following discussion.

The parameters used in the NORAD element set (elset) are:

Epoch time: As used by NORAD in a general perturbations element set the epoch time of a satellite is taken as the time of passage through the equatorial plane on the last ascending pass of the satellite.

Inclination: Inclination is defined as the angle made between the satellite's orbital plane and the equatorial plane in the direction of satellite movement at the ascending node.

Ascending node: The ascending node is the point of passage of the satellite's orbit through the equatorial plane in a south to north direction. It is measured in degrees right ascension for the first point of Aries.

Eccentricity: Eccentricity is a measure of the flatness of an orbit since the orbit is in the shape of an ellipse.

Argument of perigee: The argument of perigee is the angular distance between the ascending node and the perigee of the orbit in the direction of satellite motion.

Mean anomaly: Mean anomaly is a measure of the angular distance between the argument of perigee and the satellite's position in the orbit, in the direction of satellite motion, at the epoch time. In the NORAD general perturbations element set the mean anomaly and argument of perigee must add to 360 degrees.

Mean motion (n): Mean motion is a representation of a satellite's orbital period. It is not, however, expressed in the time to complete one revolution, but in terms of revolutions per day. Its relation to period is given by $n = 1440/P$, where P is in minutes.

$\dot{N}/2$ ($\dot{N}/2$): $\dot{N}/2$ is a term given with the NORAD elset as the difference between a Brouwer and a Kozai mean motion (1), where Kozai used a fourth order general perturbations model and Brouwer used a simplified model of a satellite's orbit. It estimates orbital decay rate.

BSTAR: BSTAR is a synthetic drag term for use in the equations of motion. It is a comparison of the orbit of a satellite to some standard reference satellite. It does not account for size or shape differences or solar flux. Its prime use is to indicate which satellites have been more affected by changes in solar flux. There is little other practical use for this term.

Figure 2.1 shows some typical NORAD 2-card elsets.

Three other parameters are used in this thesis, though neither is in a normal elset. One of them is \dot{P} , which is the rate of change of orbital period per day. It is provided by the Naval Space Surveillance Center (NAVSPASUR). Another parameter used in this thesis is \dot{n} . For the purpose of this thesis \dot{n} will be estimated as the difference between n at two different times, divided by the difference in time

such that:

$$\dot{n}(t_0) = \frac{n(t_1) - n(t_0)}{t_1 - t_0} \quad (2.1)$$

The final parameter is \ddot{n} , which, in this thesis, is the difference in n at two times divided by the delta time so:

$$\ddot{n}(t_0) = \frac{\dot{n}(t_1) - \dot{n}(t_0)}{t_1 - t_0} \quad (2.2)$$

The parameters $N/2$, \dot{n} , \ddot{n} will be used in this thesis to estimate new values of n .

```

01-PP-05      12:01:00      1.0000000000000000
1 148241 81 11 " 85064.65010458 .000000000 00000-0 15785-3 0 02870
2 14824 27.1278 239.0351 0562080 69.7509 206.7249 14.582100005167070

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Figure 2.1 NORAD 2-card element set

The Format of the 2-card elset is:

Line 1: line number, satellite number, international designator,
epoch time, $\dot{N}/2$, $\ddot{N}/6$, Bstar, elset number.

Line 2: line number, satellite number, inclination, right ascension,
eccentricity, argument of perigee, mean anomaly, mean motion.

ORBITAL MECHANICS

A satellite in near earth orbit is affected by the upper reaches of the atmosphere. The satellite has a velocity of about 7 km/s relative to the upper atmosphere. As in the movement of any body through a medium, there will be a retarding force on the body opposite to the direction of motion. This retarding force is called drag and is given by:

$$\vec{D} = - \frac{C_p A v^2 \vec{v}}{2v} \quad (2.3)$$

where \vec{D} = drag force

C = drag coefficient

p = atmospheric density

A = satellite cross sectional area

\vec{v} = relative vehicle velocity (3-21)

This will give an acceleration of the satellite such that:

$$\ddot{\vec{r}} = - \frac{C A p v \dot{\vec{r}}}{2m} \quad (2.4)$$

where $\ddot{\vec{r}}$ = acceleration due to drag

m = mass of the satellite

$\dot{\vec{r}}$ = total satellite velocity (4-423)

These equations can be combined with the standard equations of motion and integrated to find the satellites orbit. The largest obstruction to this is finding the correct value(s) of p to use in the model. Both altitude and solar flux affect p and must be accounted for.

CURRENT MODELS

Satellite decay is covered by several models. These can be either estimating type models such as that developed in this thesis, or atmospheric density models which compute values of p for use in the equations of motion. Each type of model has advantages and disadvantages. If accuracy is required and values of solar flux are known, then the atmospheric density models are best. If solar flux values are unknown or long time estimates are needed, estimating models are more suitable. The exact choice of model must be made in view of the product desired.

Of the atmospheric density models, one of the most useful to date is the Jacchia atmospheric model. There are several versions of the Jacchia atmosphere model. One of them, the J65 model produces the density (p) values through a table "look up" method. It uses the values of the solar flux to find density values in a table. It relates values of the solar flux at a wavelength of 10.7 cm to density given in a table compiled by Nicolet (2-B14). It should be noted that the 10.7 cm flux ($F_{10.7}$) does not heat the upper atmosphere. The upper atmosphere is heated by the extreme ultra violet (EUV) radiation which cannot be observed at the earth's surface, whereas the $F_{10.7}$ flux can be measured (2-B16). The J70 model uses both an average value of $F_{10.7}$ taken at a time of $t - 400$ days and the changing daily value for a time lag of one day such that:

$$T = 383 + 3.3\overline{F} + 1.8(F - \overline{F}) \quad (2.5)$$

where \bar{F} = average value of F10.7 flux at t-400 days

F = previous days value of F10.7 flux

T = temperature (2-B17)

The J70 model can then calculate the density through suitable manipulation of thermodynamic gas laws.

Other models produce the p values in different ways. For example, the DENSEL model uses a power function in altitude and theoretical values of F10.7 (2-B14,B16) to find p. There is also the exponential atmosphere:

$$p = p_0 \exp[-c(h - h_0)] \quad (2.6)$$

where p_0 = density at height h_0

c = a constant (2-B13)

The exponential atmosphere is similar to the model used by Desmond King-Hele to determine satellite lifetimes, where King-Hele uses:

$$p = p_0 \exp[-(y - y_0)/H] \quad (2.7)$$

where p_0 = density at y_0

H = scale height (5-182)

and then:

$$z = ae/H \quad (2.8)$$

where a = semi major axis

e = orbital eccentricity (5-182)

so that the satellite lifetime L may be written as:

$$L = \frac{3en}{4n} \left[\frac{1+7e+5e^2}{6} + \frac{1}{16} \left(\frac{1+11e+3e^2}{12} + \frac{3}{4z} + \frac{3}{4z^2} \right) + 0(e, 0.5/z) \right] \quad (5-182) \quad (2.9)$$

Any of the models that calculate density are accurate for the time calculated, but are valid for only a short time before the orbit changes beyond prediction limits. This is due to uncertainty of future solar flux, and, therefore, atmospheric density values. Another problem with this type of model is the calculation time required, on the order of 31 seconds to produce the densities (2-3), and minutes to hours to integrate the equations of motion for each satellite. This quickly grows beyond acceptable limits when dealing with many satellites.

A method for predicting long term changes is in use by Naval Space Surveillance (NAVSPASUR). This method is based on calculating values of PDOT. PDOT is the change in period per day of a satellites orbit. It is not highly accurate, but offers two advantages:

- 1) It gives a general indication of a satellites period over very long time spans.
- 2) It is very quick, requiring less than a minute on a hand-held calculator. The equation used by this method is:

$$P(t) = P(0) + \dot{P}t \quad (2.10)$$

This method is used by NAVSPASUR and NORAD to determine if a lost satellite may have decayed. Because of the inaccuracies, the satellite is not counted as decayed unless the period has been below 87.5 minutes (the standard cutoff for an orbit) for over a month, and there are no possible unknown satellites between its old position and possible current positions. In practice the use of this method is more an art

than a science. It is only used by the most qualified analysts because a great deal of judgment on the part of the analyst is required.

A third method, used mostly by NORAD, is based upon NDOT. NDOT is the difference between a Brouwer and a Kozai mean motion for the same time (1). Brouwer and Kozai are two methods of calculating mean motion used in the NORAD system. The method is very similar to the PDOT except that the change is calculated for mean motion instead of period. In application the equation used is:

$$n(t) = n(0) + 2(\dot{N}/2)dt \quad (2.11)$$

Like the PDOT, the NDOT requires experience to use and produces similar results.

The primary model developed in this thesis is based upon the NDOT model. It is employed in much the same manner as described above but includes an allowance for solar flux and error boundaries on the accuracy of the calculated value of n . This provides a measure of the uncertainty in predicting a new value for n . Solar flux is addressed by calculating an average $\dot{N}/2$ from several points out of a 30 day interval since solar flux varies over about a 27 day interval (6-189).

MEASURES OF EFFECTIVENESS

The accuracy limits are determined to allow a reasonable prediction at 100 days and still have a recoverable satellite. From experience this was set at about 0.1 min/rev which gives an error limit of 0.01595 rev/day (for a 95 minute orbit). This is an error range of 1.58 revs at 100 days.

III. METHODOLOGY

DATA

The data used in this thesis was provided by NORAD/HDS. The data consisted of 50 days of element sets on a group of 5 random satellites chosen from a set of 70 satellites with mean motions between 16 and 14 rev/day. The satellites were chosen at random and were not given individual identifiers to eliminate the possibility of selecting data with some bias. The following data on each of the 5 satellites was collected: time, delta time, $\dot{n}(t)$, $\dot{n}(t-dt)$, $n(t)$, $n(t=0)$.

In addition to these data, the values of F10.7 solar flux for the epoch date, the change in F10.7 flux from the previous day and the change in F10.7 flux on the previous day were added. This information was placed in a file on the Cyber computer. Appendix A contains a listing of this file. The data are in the format:

$$t, dt, \dot{n}(t), \dot{n}(t-dt), n(t), n(0), S, F, SF.$$

The values of \dot{n} were calculated by equation 2.1 and placed in the file along with the data transferred from the NORAD elsets.

The data in this file were then used in the program BMDP to fit the models to the data.

MODEL DEVELOPMENT

A model for n may be developed in several ways. One of the models used at NORAD involves a Taylor series expansion of n to give:

$$n(t) = n(0) + \dot{n}(t - t_0) + \frac{1}{2}\ddot{n}(t - t_0)^2 + \dots \quad (3.1)$$

where NORAD defined \dot{n} as $2(\dot{N}/2)$ and \ddot{n} as $3(\ddot{N}/6)$ so that:

$$n(t) = n(0) + 2(\dot{N}/2) (t - t_0) + 3(\ddot{N}/6) (t - t_0)^2 + \dots \quad (3.2)$$

and $t - t_0$ is the time of interest minus the initial time. The second and higher order terms are of small magnitude compared to n for the normal values of $(t - t_0)$ encountered in element propagation. Therefore the second and higher order terms are set to zero. In normal usage the equation used for n is:

$$n(t) = n(0) + 2(\dot{N}/2) (t - t_0) \quad (3.3)$$

That this formula is appropriate can be seen from figure 3.1 which is a plot of mean motion versus time. This figure shows that the value of n at any one time depends on the value of n at some previous time plus the sum of all the changes in n over the time difference. The sum of all changes can also be considered as an integral over all changes. Under the assumption that all changes over time greater than second order are very small and can be considered zero, the second order change, \ddot{n} , can be considered a constant. That is:

$$\ddot{n} = a \quad (3.4)$$

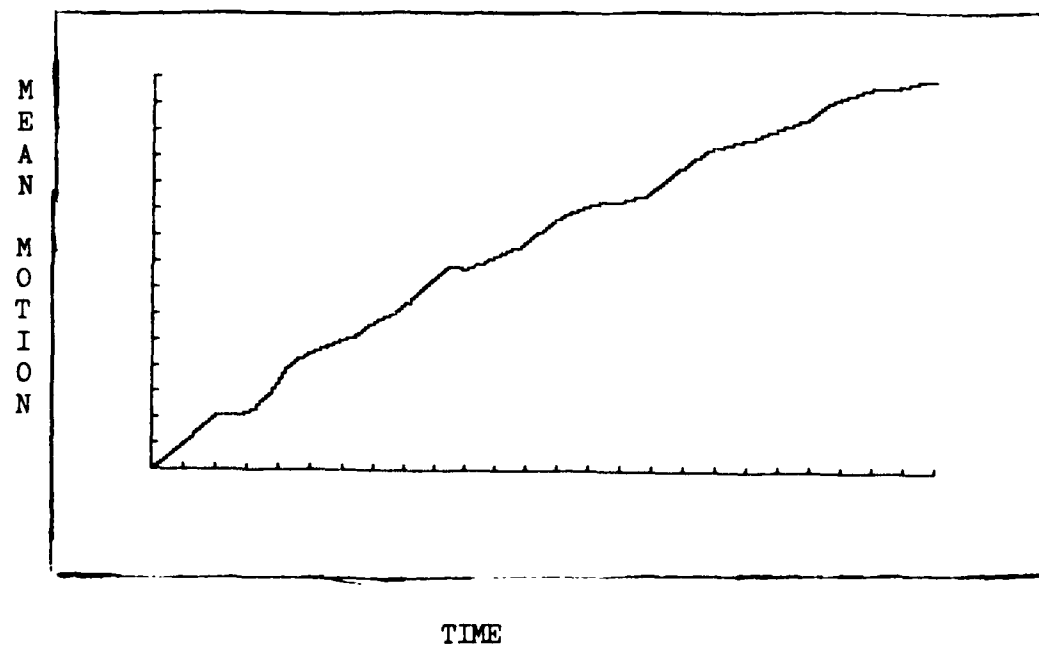


Figure 3.1 Typical Mean Motion for days 84300 through 85084

And since:

$$\ddot{n} = \frac{d^2 n}{dt^2} = \frac{d\dot{n}}{dt} \quad (3.5)$$

This gives:

$$\frac{d\dot{n}}{dt} = a \quad (3.6)$$

Which may be written as:

$$d\dot{n} = a dt \quad (3.7)$$

Equation 3.7 may be integrated:

$$\int d\dot{n} = a \int dt \quad (3.8)$$

$$\dot{n} - \dot{n}_0 = a(t - t_0) \quad (3.9)$$

$$\dot{n} = \dot{n}_0 + a(t - t_0) \quad (3.10)$$

And since:

$$\dot{n} = \frac{dn}{dt} \quad (3.11)$$

$$\frac{dn}{dt} = \dot{n}_0 + a(t - t_0) \quad (3.12)$$

$$dn = \left[\dot{n}_0 + a(t - t_0) \right] dt \quad (3.13)$$

Integrating equation 3.13 gives:

$$\int dn = \int [\dot{n}_0 + a(t - t_0)] dt \quad (3.14)$$

$$n - n_0 = \dot{n}_0(t - t_0) + \frac{1}{2}a(t - t_0)^2 \quad (3.15)$$

$$n = n_0 + \dot{n}_0(t - t_0) + \frac{1}{2}a(t - t_0)^2 \quad (3.16)$$

Substituting in equation 3.4 lead to an equation of the form:

$$n(t) = n(0) + \dot{n}(t - t_0) + \frac{1}{2}\ddot{n}(t - t_0)^2 \quad (3.17)$$

This equation is basically the same as that given by a Taylor series, but is arrived at from physical considerations.

From these come two of the models that will be studied in this thesis. Equation 3.17 is the baseline n0 model and is used in BMDP in modified form. Equation 3.3 is the standard NORAD model (herein called model n5).

Model n0 appears in this thesis in three basic forms. It appears in its original version as given by equation 3.17. There are also two modified versions which are used in BMDP. They include model n1 which included regression coefficients in each term and has the intercept defined at $B_1 n(0)$ such that:

$$n1: \quad n(t) = B_1 n(0) + B_2 \dot{n}(t - t_0) + 0.5 B_3 \ddot{n}(t - t_0)^2 \quad (3.18)$$

where the B's are the regression coefficients from BMDP. Next, under the assumption that the regression coefficients contain the physical

constants and the solar flux dependence, and that long term average solar effects are contained in the intercept since over time flux approaches a constant gives:

$$n_2: \quad n(t) = B_0 + B_1 n(0) + B_2 \dot{n}(t - t_0) + B_3 \ddot{n}(t - t_0)^2 \quad (3.19)$$

Next, assuming there is no $(t - t_0)^2$ dependence, that is, the satellites "forgets" earlier changes such that there is no second order time dependence, gives model n_3 :

$$n_3: \quad n(t) = B_0 + B_1 n(0) + B_2 \dot{n}(t - t_0) + B_3 \ddot{n}(t - t_0) \quad (3.20)$$

The final model comes from assuming a simple linear relation between the terms. Model n_4 is:

$$n_4: \quad n(t) = B_0 + B_1(t - t_0) + B_2 \dot{n} + B_3 n(0) + B_4 F \quad (3.21)$$

Note that only model n_4 contains solar flux explicitly, where it is given as the change in the flux for time $t-1$. Solar flux changes are contained implicitly within the regression coefficients and the \dot{n} and \ddot{n} terms in models n_1 , n_2 , and n_3 . The reason for this can be seen by comparison of figures 3.1 and 3.2, where figure 3.2 is a plot of solar flux for the same time frame as figure 3.1. It is obvious from the graph that the daily solar flux value has little long term effect on individual values of n .

Likewise, a comparison of figure 3.2 and figure 3.3 (which is a graph of the daily change in n , i.e. \dot{n}) shows a much higher correlation between \dot{n} and flux. That this is indeed the case will be shown by the data analysis carried out in chapter four.

None of the models contains orbital elements other than those related to mean motion. Eccentricity is not included because it is near zero for the orbits being studied. While it is known that atmospheric density is not constant around an orbit (density changes with latitude and darkness), it is assumed in this thesis that effects relating to inclination and other elements are very small for time spans on the order of 100 days. Their inclusion in a model is an area for additional study.

Below is given a complete list of the models that will be used in the analysis by BMDP, where t is now defined as $t - t_0$.

$$n0: \quad n = n_0 + \dot{n}t + 0.5\ddot{n}t^2 \quad (3.17)$$

$$n1: \quad n = B_1 n_0 + B_2 \dot{n}t + 0.5B_3 \ddot{n}t^2 \quad (3.18)$$

$$n2: \quad n = B_0 + B_1 n_0 + B_2 \dot{n}t + B_3 \ddot{n}t^2 \quad (3.19)$$

$$n3: \quad n = B_0 + B_1 n + B_2 \dot{n}t + B_3 \ddot{n}t \quad (3.20)$$

$$n4: \quad n = B_0 + B_1 t + B_2 \dot{n} + B_3 n_0 + B_4 F \quad (3.21)$$

$$n5: \quad n = n + 2(\dot{N}/2)t \quad (3.3)$$

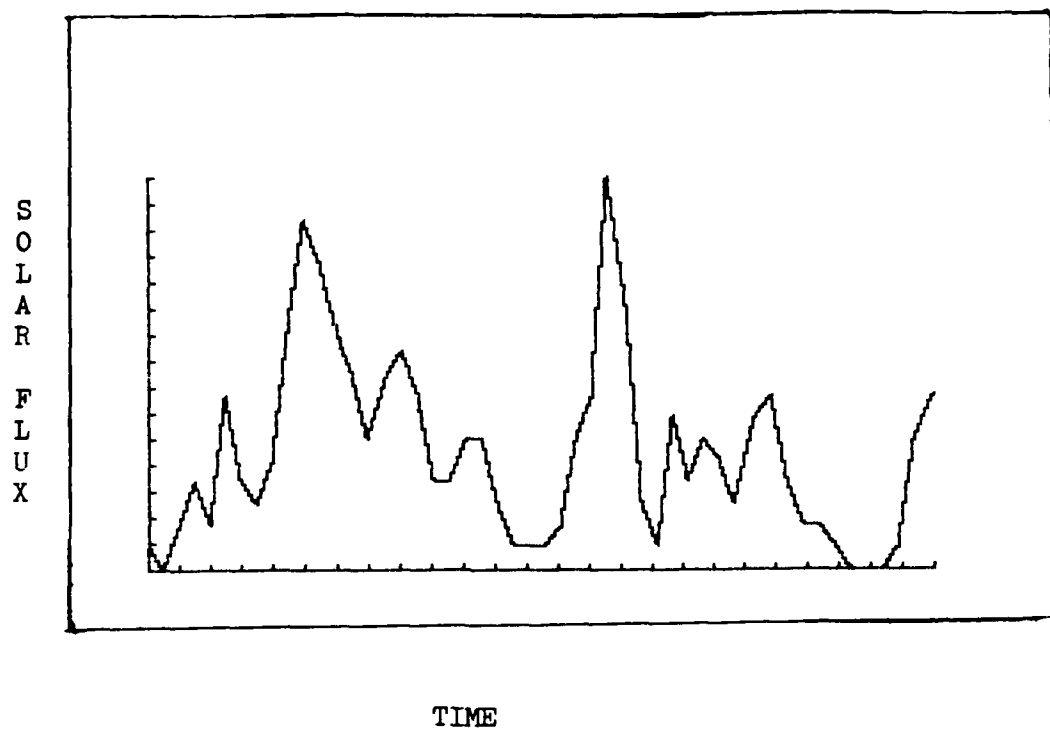


Figure 3.2 Solar Flux for days 84300 through 85084

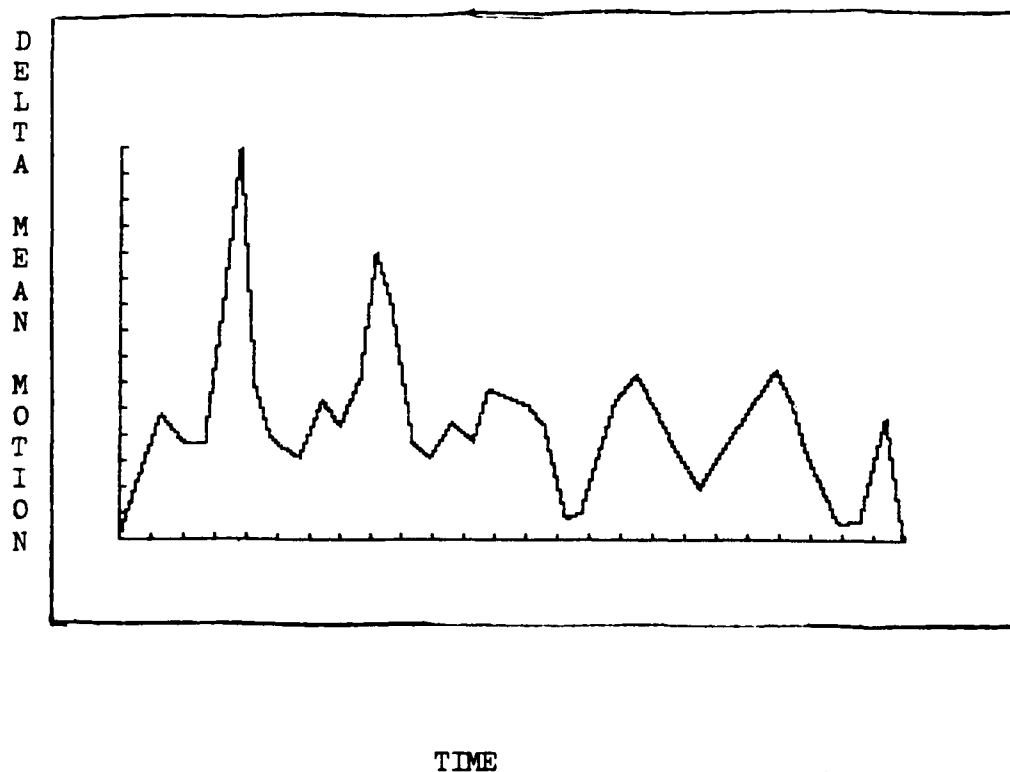


Figure 3.3 Daily Change in Mean Motion for days 84300 through 85084

IV. RESULTS

SOLAR FLUX

In chapter three it was stated that solar flux was not a major explicit factor in predicting values of n . Some justification, on the basis of graphical analysis, was given. Further empirical justification will be given here.

In the BMDP analysis of model n_4 it is shown that there is only a 0.0122 correlation between F and n (see Table 4.4), while the regression coefficient is of the order of $10E-4$. This shows that flux has little effect on n directly. A separate analysis shows that there is a -0.1021 correlation between F and \dot{n} .

Also, if flux is considered as a cumulative average and the daily change from that average, it is seen that the average rapidly approaches a constant plus or minus the daily change. If this constant is included in the regression equations, it must be in the value of the intercept constant. The daily change from a constant will be in the daily values of the second and higher order terms of the n equation. That is, since for a nearly circular orbit there is no altitude dependence for drag, the rate of decay will contain a constant value for the average density at that altitude. This constant is the first order term, \dot{n} . The rate of change of \dot{n} , \ddot{n} will be constant for a constant change in solar flux. Since flux is not constant on a daily basis, there will be a change in \ddot{n} , given by the third and higher order terms, which are nearly zero and are defined as zero such that \ddot{n} is a constant of small magnitude. Therefore the statement that the solar flux dependence is

implicit in n and the regression coefficients is supported.

BMDP ANALYSIS

The models included in the BMDP analysis were:

$$n1: \quad n = B_1 n_o + B_2 \dot{n}t + 0.5 B_3 \ddot{n}t^2 \quad (3.18)$$

$$n2: \quad n = B_o + B_1 n_o + B_2 \dot{n}t + B_3 \ddot{n}t^2 \quad (3.19)$$

$$n3: \quad n = B_o + B_1 n_o + B_2 \dot{n}t + B_3 \ddot{n}t \quad (3.20)$$

$$n4: \quad n = B_o + B_1 t + B_2 \dot{n} + B_3 n_o + B_4 F \quad (3.21)$$

BMDP was run against the data file in Appendix A using each of these models independently. The output from BMDP produced the results shown in tables 4.1 through 4.4.

TABLE 4.1

BMDP RESULTS FOR MODEL N1

$$\text{Model} \quad n1: \quad n = B_1 n_o + B_2 \dot{n}t + 0.5 B_3 \ddot{n}t^2 \quad (3.18)$$

Multiple R=1 $R^2=1$ standard error=0.0118

$$B_1=1 \quad B_2=1.06457 \quad B_3=0.01970$$

$$n = n_o + 1.06457 \dot{n}t + (0.0197/2) \ddot{n}t^2$$

	Sum of Squares	DF	Mean Square	Ratio	P
Regression	22260.2993	3	7420.0998	53696658.325	0.0
Residual	0.0130	94	0.0001		

	n_o	\dot{n}	\ddot{n}	$\dot{n}t$	$\ddot{n}t^2$
Correlation with n	0.9962	0.8274	0.1138	0.7135	0.1118

TABLE 4.2

BMDP RESULTS FOR MODEL N2

Model	n2: $n = B_0 + B_1 n_0 + B_2 \dot{n}t + B_3 \ddot{n}t^2$					(3.19)

Multiple R=0.9992	$R^2=0.9983$		standard error=0.0115			
$B_0=-0.20596$	$B_1=1.01384$	$B_2=0.98394$	$B_3=0.00935$			
$n=-0.20596+1.01384n_0+0.98394\dot{n}t+0.00935\ddot{n}t^2$						

	Sum of Squares	DF	Mean Square	Ratio	P	
Regression	7.3171	3	2.4390	18347.738	0.000	
Residual	0.0124	93	0.0001			

	n_0	\dot{n}	\ddot{n}	$\dot{n}t$	$\ddot{n}t^2$	
Correlation	-----					
with n	0.9962	0.8274	0.1138	0.7135	0.1118	

TABLE 4.3

BMDP RESULTS FOR MODEL N3

Model n3: $n = B_0 + B_1 n_0 + B_2 \dot{n}t + B_3 \ddot{n}t$ (3.20)

Multiple R=0.9992 $R^2=0.9983$ standard error=0.0114

$B_0=-0.18935$ $B_1=1.01273$ $B_2=0.99835$ $B_3=0.43515$

$n=-0.18935+1.01273n_0+0.99835\dot{n}t+0.43515\ddot{n}t$

	Sum of Squares	DF	Mean Square	Ratio	P
Regression	7.3174	3	2.4391	18659.405	0.00
Residual	0.0122	93	0.0001		

	n_0	\dot{n}	\ddot{n}	$\dot{n}t$	$\ddot{n}t$
Correlation with n	0.9962	0.8274	0.1138	0.7135	0.1089

TABLE 4.4

BMDP RESULTS FOR MODEL N4

Model	$n_4: n = B_0 + B_1 t + B_2 \dot{n} + B_3 n_0 + B_4 F$				(3.21)
<hr/>					
Multiple	R=0.9981	$R^2=0.9962$	standard error=0.0174		
$B_0=-0.67573$	$B_1=0.00090517$	$B_2=17.45671$	$B_3=1.04379$		
$B_4=-0.000097511$					
$n=-0.67573+(9.0517E-4)t+17.45671\dot{n}+1.04379n_0+(-9.7511E-5)F$					
<hr/>					
	Sum of Squares	DF	Mean Square	Ratio	P
<hr/>					
Regression	7.7315	4	1.9329	6366.816	0.000
Residual	0.0294	97	0.0003		
<hr/>					
	t	\dot{n}	n_0	F	
<hr/>					
Correlation with n	-0.0023	0.8160	0.9963	0.0122	

POST BMDP ANALYSIS

The equations produced by BMDP, along with n_0 and n_5 , were used to generate predicted n values for satellites 15363, 14476, 13043, and 7840. The predictions were done for time spans of 10, 30, 50, and 100 days (as close to these values as possible given the NORAD data). The calculated value of n was subtracted from the actual value of n at each time to give a value for the error in n such that a negative error indicates that the calculated value was greater than the actual value at time t . Tables 4.5 through 4.11 contain the sample data for these four satellites for each model. The values on \dot{n} and \ddot{n} were calculated by equations 2.1 and 2.2, and are shown in Table 4.5.

TABLE 4.5
VALUES OF \dot{n} , \ddot{n} , AND $\dot{N}/2$ FOR EACH SATELLITE

SATELLITE	\dot{n}	\ddot{n}	$\dot{N}/2$
15363	0.00061318	0.00007924	0.00033037
14476	0.00002221	0.00000051	0.00001155
13043	0.00049423	0.00001846	0.00027705
7840	0.00002121	0.00000222	0.00001401

TABLE 4.6

MEAN MOTION VALUES FOR MODEL NO

SATELLITE	t days	ACTUAL n rev/day	CALCULATED n rev/day	ERROR rev/day
15363	0	14.61592219	-----	-----
	10	14.62055640	14.6260108	-0.00545440
	30	14.62850624	14.6699376	-0.04143136
	50	14.63611823	14.7455305	-0.10941227
	100	14.65988312	15.0730518	-0.41316868
14476	0	14.47940319	-----	-----
	10	14.47959223	14.4796510	-0.00005877
	31	14.48005842	14.4803389	-0.00028048
	52	14.48061500	14.4812537	-0.00063870
	102	14.48162641	14.4843450	-0.00271859
13043	0	15.66287477	-----	-----
	10	15.66745489	15.6687299	-0.00128501
	30	15.67900804	15.6860070	-0.00699896
	50	15.68914747	15.7106564	-0.02150893
	100	15.71437096	15.8045781	-0.09020714
7840	0	14.56440052	-----	-----
	10	14.56459216	14.5647237	-0.00013154
	29	14.56491839	14.5659499	-0.00103151
	52	14.56555106	14.5685076	-0.00295654
	103	14.56649157	14.5783721	-0.01188053

TABLE 4.7

MEAN MOTION VALUES FOR MODEL N1

SATELLITE	t days	ACTUAL n rev/day	CALCULATED n rev/day	ERROR rev/day
15363	0	14.61592219	-----	-----
	10	14.62055640	14.6225264	-0.00197000
	30	14.62850624	14.6362026	-0.00769626
	50	14.63611823	14.6505026	-0.05286367
	100	14.65988312	14.6889819	-0.02909878
14476	0	14.47940319	-----	-----
	10	14.47959223	14.4796401	-0.00004787
	31	14.48005842	14.4801410	-0.00008258
	52	14.48061500	14.4806463	-0.00003130
	102	14.48162641	14.4818675	-0.00024109
13043	0	15.66287477	-----	-----
	10	15.66745489	15.6681544	-0.00069951
	30	15.67900804	15.6788227	0.00018534
	50	15.68914747	15.6896365	-0.00048903
	100	15.71437096	15.7173072	-0.00293624
7840	0	14.56440052	-----	-----
	10	14.56459216	14.5646204	-0.00002824
	29	14.56491839	14.5650397	-0.00012131
	52	14.56555106	14.5655565	-0.00000551
	103	14.56649157	14.5668036	-0.00031203

TABLE 4.8

MEAN MOTION VALUES FOR MODEL N2

SATELLITE	t days	ACTUAL n rev/day	CALCULATED n rev/day	ERROR rev/day
15363	0	14.61592219	-----	-----
	10	14.62055640	14.6183525	0.00220390
	30	14.62850624	14.6310084	-0.00250216
	50	14.63611823	14.6442566	-0.00813837
	100	14.65988312	14.6799675	-0.01979188
14476	0	14.47940319	-----	-----
	10	14.47959223	14.4745202	0.00553513
	31	14.48005842	14.4745202	0.00553822
	52	14.48061500	14.4749875	0.00562750
	102	14.48162641	14.4751171	0.00550931
13043	0	15.66287477	-----	-----
	10	15.66745489	15.6785692	-0.01111431
	30	15.67900804	15.6884331	-0.00942506
	50	15.68914747	15.6984351	-0.00928763
	100	15.71437096	15.7240441	-0.00967314
7840	0	14.56440052	-----	-----
	10	14.56459216	14.5602155	0.00437666
	29	14.56491839	14.5606034	0.00431499
	52	14.56555106	14.5610904	0.00446066
	103	14.56649157	14.5622384	0.00425317

TABLE 4.9

MEAN MOTION VALUES FOR MODEL N3

SATELLITE	t days	ACTUAL n rev/day	CALCULATED n rev/day	ERROR rev/day
15363	0	14.61592219	-----	-----
	10	14.62055640	14.6190976	0.00145880
	30	14.62850624	14.6320271	-0.00352086
	50	14.63611823	14.6449565	-0.00883827
	100	14.65988312	14.6772802	-0.01739708
14476	0	14.47940319	-----	-----
	10	14.47959223	14.4746000	0.00492230
	31	14.48005842	14.4750703	0.00498812
	52	14.48061500	14.4755406	0.00050744
	102	14.48162641	14.4766604	0.00496601
13043	0	15.66287477	-----	-----
	10	15.66745489	15.6779276	-0.01047271
	30	15.67900804	15.6879566	-0.00894796
	50	15.68914747	15.6979856	-0.00883813
	100	15.71437096	15.7230580	-0.00868704
7840	0	14.56440052	-----	-----
	10	14.56459216	14.5606685	0.00392366
	29	14.56491839	14.5610645	0.00385389
	52	14.56555106	14.5615438	0.00400726
	103	14.56649157	14.5620670	0.00442457

TABLE 4.10

MEAN MOTION VALUES FOR MODEL N4

SATELLITE	t days	ACTUAL n rev/day	CALCULATED n rev/day	ERROR rev/day
15363	0	14.61592219	-----	-----
	10	14.62055640	14.5997121	0.02084430
	30	14.62850624	14.6182269	0.01027934
	50	14.63611823	14.6361425	-0.00002427
	100	14.65988312	14.6812678	-0.05138468
14476	0	14.47940319	-----	-----
	10	14.47959223	14.4470907	0.03250153
	31	14.48005842	14.4662946	0.01376382
	52	14.48061500	14.4851002	-0.00448520
	102	14.48162641	14.5303213	-0.04869489
13043	0	15.66287477	-----	-----
	10	15.66745489	15.6907502	-0.03004711
	30	15.67900804	15.7087211	-0.02971306
	50	15.68914747	15.7269894	-0.03784193
	100	15.71437096	15.7722919	-0.05792094
7840	0	14.56440052	-----	-----
	10	14.56459216	14.5358578	0.02873436
	29	14.56491839	14.5529297	0.01198869
	52	14.56555106	14.5740243	-0.00847324
	103	14.56649157	14.6201866	-0.05369503

TABLE 4.11

MEAN MOTION VALUES FOR MODEL N5

SATELLITE	t days	ACTUAL n rev/day	CALCULATED n rev/day	ERROR rev/day
15363	0	14.61592219	-----	-----
	10	14.62055640	14.6225296	-0.00198320
	30	14.62850624	14.6357444	-0.00723816
	50	14.63611823	14.6489592	-0.01284097
	100	14.65988312	14.6819962	-0.02211308
14476	0	14.47940319	-----	-----
	10	14.47959223	14.4796342	-0.00004197
	31	14.48005842	14.4801193	-0.00006088
	52	14.48061500	14.4806044	0.00001060
	102	14.48162641	14.4817594	-0.00013299
13043	0	15.66287477	-----	-----
	10	15.66745489	15.6684158	-0.00096091
	30	15.67900804	15.6794978	-0.00048976
	50	15.68914747	15.6905798	-0.00143233
	100	15.71437096	15.7182848	-0.00391384
7840	0	14.56440052	-----	-----
	10	14.56459216	14.5646807	-0.00021484
	29	14.56491839	14.5652131	-0.00029471
	52	14.56555106	14.5658576	-0.00030654
	103	14.56649157	14.5672866	-0.00079503

Since the criteria for quality of a model was set at 0.01595 rev/day at 100 days, the absolute value of the error at 100 days (as near to 100 days as the data allowed) was averaged over the four satellites for each model. This average error was used to measure the model quality and compare the models to each other. Table 4.12 compares the 100 day error values of all satellites and all models.

TABLE 4.12

100 DAY ERROR ABSOLUTE VALUES FOR ALL SATELLITES AND MODELS

MODEL/SATELLITE	15363	14476	13043	7840
n0:	0.41316868	0.00271859	0.09020714	0.01188053
n1:	0.02909878	0.00024109	0.00293624	0.00031203
n2:	0.01979188	0.00550931	0.00967314	0.00425317
n3:	0.01739708	0.00496601	0.00868704	0.00442457
n4:	0.05138468	0.04869489	0.05792094	0.05369503
n5:	0.02211308	0.00013299	0.00391384	0.00079503

Table 4.13 shows the average of the 100 day error values for each model in order from smallest to largest.

TABLE 4.13

AVERAGE 100 DAY ERROR VALUES FOR EACH MODEL

MODEL	ERROR rev/day
n5	0.0067387
n1	0.0081470
n3	0.0088687
n2	0.0098068
n4	0.0529239
n0	0.1295098

Analysis of the data for each model indicates the following:

- 1) Models n0 and n4 have a 100 day error greater than 0.01595 rev/day and are removed from consideration.
- 2) Models n1, n2, n3, and n5 all have acceptable 100 day error values as shown in table 4.14.

TABLE 4.14

AVERAGE 100 DAY ERROR VALUES FOR ACCEPTABLE MODELS

MODEL	ERROR rev/day
n5	0.0067387
n1	0.0081470
n3	0.0088687
n2	0.0098068

3) Model n5 is the best in terms of 100 day error. It also produces very accurate values of n at times other than 100 days.

4) Models n2 and n3 are very consistent in the magnitude of the error for any satellite as shown in Tables 4.8 and 4.9. This is shown in figures 4.1 and 4.2 which are plots of actual and calculated values on n .

In studying the data it was found that the greatest error, for example for satellite 15363, occurred when \dot{n} was unstable. To eliminate this, $\dot{N}/2$ for satellite 15363 was averaged over a 30 day span using 10 day intervals. This decreased the 100 day error for that satellite by a factor of seven in the n5 model. The 30 day time span is indicated by the about 27 day length of the solar cycle. Since this first test improved the results a general test was done using equations 2.1 and 2.2 for \dot{n} and \ddot{n} . The times were chosen to be at the beginning, middle, and end of the 30 day period. Averaging over this time seemed to eliminate some of the randomness in the flux dependent values. The results are shown in tables 4.15 through 4.19.

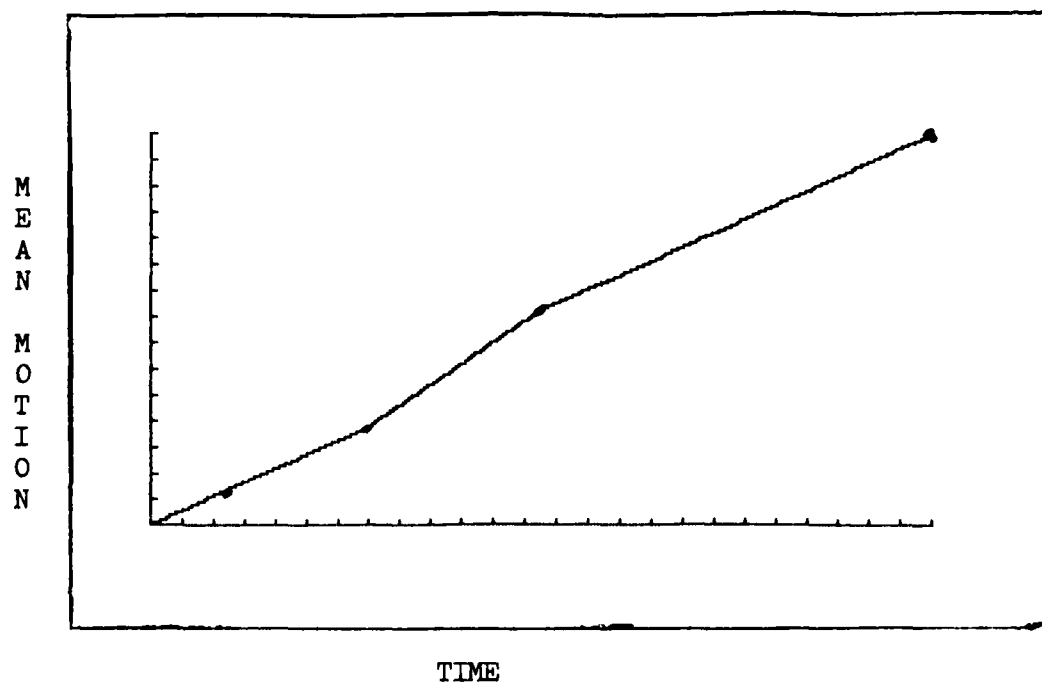


Figure 4.1 Actual and Calculated values of n for Satellite 7840 using Unaveraged \dot{n} and \ddot{n}

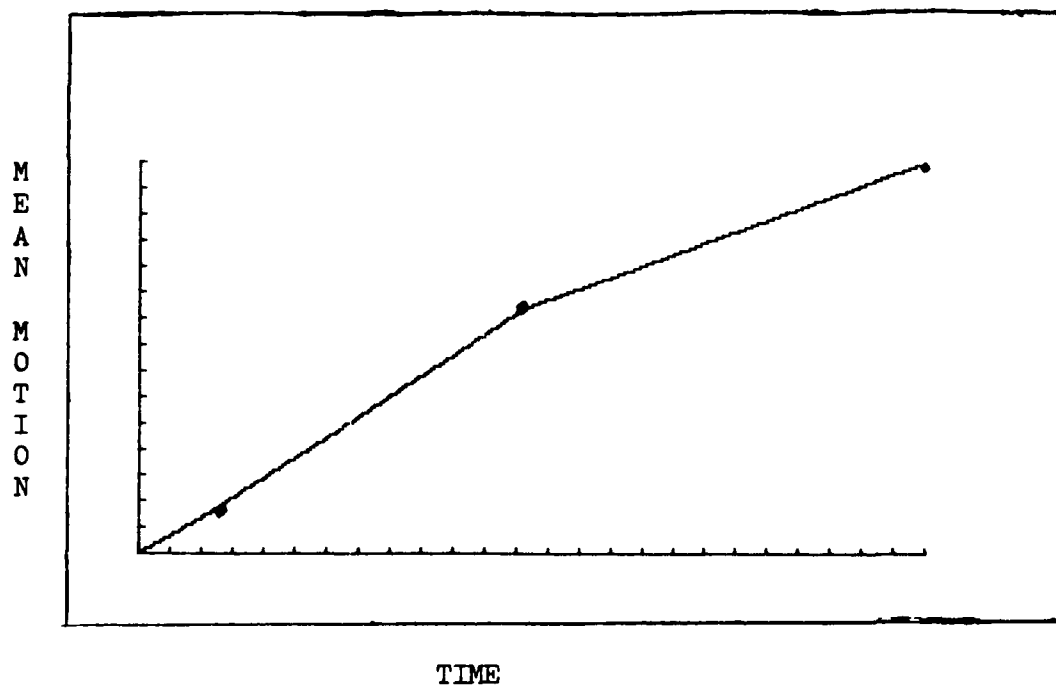


Figure 4.2 Actual and Calculated values of n for Satellite 7840 using Averaged \dot{n} and \ddot{n}

TABLE 4.15

AVERAGE VALUES OF \dot{n} AND \ddot{n} FOR EACH SATELLITE

SATELLITE	\dot{n}	\ddot{n}	$\ddot{N}/2$
15363	0.00044071	0.00000295	0.00023550
14476	0.00001892	0.00000029	0.00001156
13043	0.00049658	0.00000552	0.00026520
7840	0.00001943	0.00000022	0.00001057

TABLE 4.16

MEAN MOTION VALUES FOR MODEL N1

SATELLITE	t days	ACTUAL n rev/day	CALCULATED n rev/day	ERROR rev/day
15363	0	14.62860624	-----	-----
	10	14.63270865	14.63320080	-0.00049215
	50	14.64873275	14.65203730	-0.00330445
	100	14.67174206	14.67571370	-0.00397164
14476	0	14.48005842	-----	-----
	9	14.48028419	14.48023990	0.00004429
	51	14.48126626	14.48109410	0.00017316
	101	14.48210420	14.48212170	-0.00001750
13043	0	15.67900804	-----	-----
	11	15.68473145	15.68482970	-0.00009825
	50	15.70452036	15.70557610	-0.00105574
	100	15.73148592	15.73241580	-0.00092988
7840	0	14.56491839	-----	-----
	11	14.56516626	14.56514620	0.00002006
	52	14.56622862	14.56599960	-0.00022902
	105	14.56703165	14.56711360	-0.00008195

TABLE 4.17

MEAN MOTION VALUES FOR MODEL N2

SATELLITE	t days	ACTUAL n rev/day	CALCULATED n rev/day	ERROR rev/day
15363	0	14.62860624	-----	-----
	10	14.63270865	14.62934390	0.00336475
	50	14.64873275	14.64675540	0.00197735
	100	14.67174206	14.66864400	0.00309806
14476	0	14.48005842	-----	-----
	9	14.48028419	14.47467020	0.00561399
	51	14.48126626	14.47545890	0.00580736
	101	14.48210420	14.47641010	0.00569410
13043	0	15.67900804	-----	-----
	11	15.68473145	15.69542640	-0.01069495
	50	15.70452036	15.71460470	-0.01008434
	100	15.73148592	15.73942180	-0.00793588
7840	0	14.56491839	-----	-----
	11	14.56516626	14.56074740	0.00441886
	52	14.56622862	14.56153640	0.00469222
	105	14.56703165	14.56256640	0.00446525

TABLE 4.18

MEAN MOTION VALUES FOR MODEL N3

SATELLITE	t days	ACTUAL n rev/day	CALCULATED n rev/day	ERROR rev/day
15363	0	14.62860624	-----	-----
	10	14.63270865	14.62978980	0.00291885
	50	14.64873275	14.64744050	0.00129225
	100	14.67174206	14.66950400	0.00223806
14476	0	14.48005842	-----	-----
	9	14.48028419	14.47521070	0.00507349
	51	14.48126626	14.47600930	0.00525696
	101	14.48210420	14.47696010	0.00514410
13043	0	15.67900804	-----	-----
	11	15.68473145	15.69473160	-0.01000015
	50	15.70452036	15.71415990	-0.00963954
	100	15.73148592	15.73906790	-0.00758198
7840	0	14.56491839	-----	-----
	11	14.56516626	14.56119420	0.00397206
	52	14.56622862	14.56199330	0.00423532
	105	14.56703165	14.56302630	0.00400535

TABLE 4.19

MEAN MOTION VALUES FOR MODEL N5

SATELLITE	t days	ACTUAL n rev/day	CALCULATED n rev/day	ERROR rev/day
15363	0	14.62860624	-----	-----
	10	14.63270865	14.63221620	0.00049245
	50	14.64873275	14.65205620	-0.00332345
	100	14.67174206	14.67560620	-0.00386414
14476	0	14.48005842	-----	-----
	9	14.48028419	14.48026650	0.00001769
	51	14.48126626	14.48123750	0.00002876
	101	14.48210420	14.48239350	-0.00028930
13043	0	15.67900804	-----	-----
	11	15.68473145	15.68484240	-0.00011095
	50	15.70452036	15.70552800	-0.00100764
	100	15.73148592	15.73204800	-0.00056208
7840	0	14.56491839	-----	-----
	11	14.56516626	14.56515090	0.00001536
	52	14.56622862	14.56601770	0.00021092
	105	14.56703165	14.56713810	-0.00010645

TABLE 4.20

100 DAY ERROR ABSOLUTE VALUES FOR ALL SATELLITES AND MODELS

MODEL/SATELLITE	15363	14476	13043	7840
n1:	0.00049215	0.0000175	0.00092988	0.00008195
n2:	0.00309806	0.0056941	0.00793588	0.00446525
n3:	0.00223806	0.0051441	0.00758198	0.00400535
n5:	0.00386414	0.0002893	0.00056208	0.00010645

TABLE 4.21

AVERAGE 100 DAY ERROR VALUES FOR AVERAGED \dot{n} AND \ddot{n}

MODEL	ERROR rev/day
n5	0.0012055
n3	0.0047424
n1	0.0050009
n2	0.0052980

These values are about a factor of two (at least) improvement over the values produced by unaveraged n values. The NORAD model (n5) shows the most improvement and is still the most accurate. Its greatest problem is the instability of the error for a single satellite over time as shown in Table 4.19. In contrast, model n3, which has a small average error, shows great consistency in the magnitude of the error as shown in Table 4.18. This means that if a value of n is known at some time around the start time, the size of the error to be expected from the model can be determined for that satellite. This could be of great help in locating lost objects. For example, if an elset is available for the time at which the satellite went lost, or start time, and two to three elsets are available for ten to thirty days prior to the start time, \dot{n} and \ddot{n} may be calculated using the elset prior to the start time. Then a calculated value of n may be found and compared to the actual value of n at the start time to find a limit on the magnitude of the error. Next, at some future time, the value of n and a plus or minus range can be found. This will give the analyst general bounds within which to look for the satellite. For example satellite 519 has the following values of n at the times shown.

TABLE 4.22

SAMPLE VALUES OF MEAN MOTION

DATE	n rev/day
84312	15.0054437
84324	15.0059095
84331	15.0062630
84338	15.0063419
85072	15.0091569

Using equations 2.1 and 2.2 for times 84312, 84324, 84331 gives:

$$\dot{n} = 0.00003857 \quad \ddot{n} = 0.00000911$$

Predicting from 84331 to 84338 using model n3 gives: $n = 15.00824$

with an error of -0.0018981. Therefore the approximate error in the future is plus/minus 0.002. The actual error at 85072 is -0.0026.

The plus/minus error gives the analyst a range within which to look for the satellite.

An attempt was made to calculate \dot{n} from a simple statistical model:

$$\dot{n}(t) = B_0 + B_1 \dot{n}(t-1) + B_2 t + B_2 n_0 + B_4 F \quad (4.1)$$

This model would be used in place of equation 2.1 to find \dot{n} . BMDP was unable to produce an accurate result. The residuals from the model were of the same order of magnitude as the value being calculated. This results in no gain over conventional methods.

V. CONCLUSIONS AND RECOMMENDATIONS

The n_5 model has been shown to be the most accurate when used with 30 day average values of $\dot{N}/2$. The n_3 model is a close second to the n_5 model. Since the n_3 model is stable, if some data for n near the start time is available, limits on the error in n at some future time can be set. Because of this, the n_3 model is better for the satellite orbit types studied here, that is, low eccentricity orbits with $14 \leq n \leq 16$.

Further study is recommended in four main areas. The first area for further study is the possible use of $\dot{N}/2$ to find a value for \dot{n} and the use of these values in model n_3 . This would determine if the greater accuracy available in the n_5 model can be combined with the consistency of the n_3 model to give more accurate in-track areas in which to search for lost objects.

A second item for further study is the extension of the model to times greater than 100 days for prediction.

The third subject for study is the inclusion of mean motions outside the range included in this thesis and for more eccentric satellite orbits to be included.

The final study should determine if other orbital elements need to be included in the model.

APPENDIX A
DATA FILE USED FOR BMDP

t	dt	$\dot{n}(t)$	$\dot{n}(t - dt)$	$n(t)$	$n(\emptyset)$	S	F	SF
2	0	0.0000401	0.0000000	14.9814358	14.9813442	-1	0	70
3	1	0.0000286	0.0000401	14.9814758	14.9813442	4	-1	70
6	3	0.0000505	0.0000286	14.9815617	14.9813442	-1	4	71
9	3	0.0000558	0.0000505	14.9817133	14.9813442	0	-1	73
18	9	0.0000668	0.0000558	14.9822159	14.9813442	1	0	74
22	4	0.0000849	0.0000668	14.9825630	14.9813442	0	0	72
24	2	0.0000629	0.0000849	14.9827328	14.9813442	6	0	73
27	3	0.0000458	0.0000629	14.9829216	14.9813442	1	6	80
28	1	0.0000525	0.0000458	14.9829674	14.9813442	2	1	80
30	2	0.0000615	0.0000525	14.9830723	14.9813442	0	2	83
34	4	0.0000690	0.0000615	14.9833183	14.9813442	0	0	83
37	3	0.0000854	0.0000690	14.9835253	14.9813442	1	0	79
39	2	0.0000825	0.0000854	14.9836960	14.9813442	-2	1	80
41	2	0.0001192	0.0000825	14.9838610	14.9813442	0	-2	76
42	1	0.0000324	0.0001192	14.9839802	14.9813442	1	0	75
44	2	0.0000792	0.0000324	14.9840450	14.9813442	1	1	76
45	1	0.0000563	0.0000792	14.9841242	14.9813442	3	1	77
47	2	0.0000896	0.0000563	14.9842367	14.9813442	-1	3	81
49	2	0.0000656	0.0000896	14.9844160	14.9813442	0	-1	79
6	0	0.0000150	0.0000000	14.8682432	14.8681669	-1	0	71
16	10	0.0000208	0.0000150	14.8683929	14.8681669	-1	-1	77
22	6	0.0000334	0.0000208	14.8685179	14.8681669	0	-1	72
23	1	0.0000203	0.0000334	14.8685513	14.8681669	1	0	73
30	7	0.0000149	0.0000203	14.8686934	14.8681669	0	1	83
38	8	0.0000222	0.0000149	14.8688126	14.8681669	-2	0	79
42	4	0.0000101	0.0000222	14.8689013	14.8681669	1	-2	75
47	5	0.0000105	0.0000101	14.8689518	14.8681669	-1	1	81
48	1	0.0000146	0.0000105	14.8689623	14.8681669	-2	-1	80
2	0	0.0000141	0.0000000	14.6311455	14.6310959	-1	0	70
6	4	0.0000207	0.0000141	14.6312017	14.6310959	-1	-1	71
9	3	0.0000211	0.0000207	14.6312637	14.6310959	0	-1	73
15	6	0.0000223	0.0000211	14.6313906	14.6310959	-2	0	74
25	10	0.0000181	0.0000223	14.6316137	14.6310959	0	-2	74
29	4	0.0000134	0.0000181	14.6316862	14.6310959	2	0	81
32	3	0.0000105	0.0000134	14.6317263	14.6310959	-3	2	85

35	3	0.0000160	0.0000105	14.6317577	14.6310959	0	-3	75
39	4	0.0000141	0.0000160	14.6318216	14.6310959	-2	0	80
44	5	0.0000111	0.0000141	14.6318922	14.6310959	1	-2	76
8	0	0.0000312	0.0000000	15.0053215	15.0049667	0	0	70
12	4	0.0000238	0.0000312	15.0054464	15.0049667	1	0	72
14	2	0.0000300	0.0000238	15.0054941	15.0049667	3	1	72
16	2	0.0000243	0.0000300	15.0055542	15.0049667	-1	3	77
20	4	0.0000606	0.0000243	15.0056515	15.0049667	-3	-1	74
22	2	0.0000391	0.0000606	15.0057726	15.0049667	0	-3	72
23	1	0.0000982	0.0000391	15.0058117	15.0049667	1	0	73
24	1	0.0000458	0.0000982	15.0059891	15.0049667	6	6	73
26	2	0.0000367	0.0000458	15.0060806	15.0049667	0	6	80
28	2	0.0000591	0.0000367	15.0061541	15.0049667	2	0	80
29	1	0.0000248	0.0000591	15.0062132	15.0049667	2	2	81
31	2	0.0000264	0.0000248	15.0062628	15.0049667	1	2	85
34	3	0.0000200	0.0000264	15.0063419	15.0049667	0	1	83
36	2	0.0000267	0.0000200	15.0063820	15.0049667	0	0	79
38	2	0.0000341	0.0000267	15.0064354	15.0049667	-2	0	79
42	4	0.0000277	0.0000341	15.0065718	15.0049667	1	-2	75
46	4	0.0000404	0.0000277	15.0066824	15.0049667	-1	1	78
49	3	0.0000715	0.0000404	15.0068035	15.0049667	0	-1	79
50	1	0.0000467	0.0000715	15.0068750	15.0049667	0	0	77
1	0	0.0005770	0.0000467	15.3779602	15.3755236	0	0	70
2	1	0.0007029	0.0005770	15.3789206	15.3755236	-1	-1	70
3	1	0.0012178	0.0007029	15.3796234	15.3755236	4	-1	70
4	1	0.0012465	0.0012178	15.3808413	15.3755236	-2	4	69
5	1	0.0005617	0.0012465	15.3831081	15.3755236	0	0	73
6	1	0.0007572	0.0005617	15.3840485	15.3755236	-1	-1	71
7	1	0.0010633	0.0007572	15.3855209	15.3755236	3	3	71
8	1	0.0010376	0.0010633	15.3875751	15.3755236	0	0	70
9	1	0.0016823	0.0010376	15.3899431	15.3755236	0	0	73
10	1	0.0014992	0.0016823	15.3922539	15.3755236	-1	-1	73
11	1	0.0017462	0.0014992	15.3937531	15.3755236	-1	-1	73
12	1	0.0015879	0.0017462	15.3957367	15.3755236	1	1	72
13	1	0.0014381	0.0015879	15.3973246	15.3755236	2	1	71
14	1	0.0017490	0.0014381	15.3987627	15.3755236	3	2	72
15	1	0.0017366	0.0017490	15.4005117	15.3755236	-2	3	74
16	1	0.0016155	0.0017366	15.4022484	15.3755236	-1	-2	77
17	1	0.0017204	0.0016155	15.4046593	15.3755236	-1	-1	75

18	1	0.0013862	0.0017204	15.4063797	15.3755236	1	-1	74
20	2	0.0026684	0.0013862	15.4091520	15.3755236	-3	1	74
21	1	0.0032047	0.0026684	15.4118204	15.3755236	1	-3	75
24	3	0.0019274	0.0032047	15.4214344	15.3755236	6	1	73
25	1	0.0006914	0.0019274	15.4243202	15.3755236	0	0	74
26	1	0.0007496	0.0006914	15.4255304	15.3755236	0	0	80
27	1	0.0011072	0.0007496	15.4262800	15.3755236	1	0	80
28	1	0.0018005	0.0011072	15.4283867	15.3755236	2	2	80
29	1	0.0020504	0.0018005	15.4307737	15.3755236	2	2	81
30	1	0.0022745	0.0020504	15.4328241	15.3755236	0	2	83
31	1	0.0017424	0.0022745	15.4350986	15.3755236	1	0	85
32	1	0.0013084	0.0017424	15.4372549	15.3755236	-3	-3	85
33	1	0.0025711	0.0013084	15.4393463	15.3755236	-4	-4	86
34	1	0.0018959	0.0025711	15.4419174	15.3755236	0	-4	83
35	1	0.0021210	0.0018959	15.4438133	15.3755236	0	0	79
36	1	0.0021133	0.0021210	15.4476538	15.3755236	0	0	79
38	2	0.0016947	0.0021133	15.4548283	15.3755236	-2	-2	79
40	2	0.0012426	0.0016947	15.4597807	15.3755236	-1	-1	78
41	1	0.0012321	0.0012426	15.4616613	15.3755236	0	0	76
42	1	0.0024109	0.0012321	15.4628935	15.3755236	1	0	75
43	1	0.0017233	0.0024109	15.4653044	15.3755236	1	1	75
44	1	0.0010376	0.0017233	15.4681396	15.3755236	1	1	76
45	1	0.0005436	0.0010376	15.4779492	15.3755236	3	3	77
46	1	0.0025892	0.0005436	15.4714928	15.3755236	-1	3	78
47	1	0.0013638	0.0025892	15.4756136	15.3755236	-1	-1	81
48	1	0.0022907	0.0013638	15.4769773	15.3755236	-2	-1	80
49	1	0.0012608	0.0012608	15.4815140	15.3755236	0	0	79
50	1	0.0035906	0.0012608	15.4827747	15.3755236	0	0	77

NORAD ELEMENTS FOR SATELLITES 15363, 14476, 13043, 7840

B.1

1	153610	04110	0	05007,2013138	100013500	1000000	0	0700703	0	00000
1	153611	041075	1	05013,030000000	227,3044	137,0415	14	00000505	0	12739
1	153612	04110	0	05009,050000100	100020454	0000000	0	0411100	0	00027
2	153613	0410075	1	040,07310,0504310	214,1046	137,1058	14	000000415	0	13093
1	153614	04110	0	05011,010000000	100000000	0000000	0	0700703	0	00000
2	153615	04100	0	05013,030000000	213,0110	143,0133	14	0000715087	0	13312
1	153616	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153617	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153618	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153619	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153620	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153621	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153622	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153623	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153624	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153625	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153626	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153627	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153628	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153629	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153630	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153631	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153632	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153633	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153634	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153635	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153636	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153637	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153638	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153639	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153640	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153641	040,0075	1	040,0075,0503000	200,0112	150,0047	14	000000000	0	13620
1	153642	04110	0	05013,030000000	100020000	0000000	0	0700703	0	00000
2	153643	040,0075	1	040,0075,0503000	200,0112	150,0047	14</			

B.3

1	153041	041110	4	450747, 04409131	041110	4	450747, 04409131	040521-1	0	01312
1	153042	041110	4	450748, 04409141	041110	4	450748, 04409141	040522-1	0	01313
1	153043	041110	4	450749, 04409151	041110	4	450749, 04409151	040523-1	0	01314
1	153044	041110	4	450750, 04409161	041110	4	450750, 04409161	040524-1	0	01315
1	153045	041110	4	450751, 04409171	041110	4	450751, 04409171	040525-1	0	01316
1	153046	041110	4	450752, 04409181	041110	4	450752, 04409181	040526-1	0	01317
1	153047	041110	4	450753, 04409191	041110	4	450753, 04409191	040527-1	0	01318
1	153048	041110	4	450754, 04409201	041110	4	450754, 04409201	040528-1	0	01319
1	153049	041110	4	450755, 04409211	041110	4	450755, 04409211	040529-1	0	01320
1	153050	041110	4	450756, 04409221	041110	4	450756, 04409221	040530-1	0	01321
1	153051	041110	4	450757, 04409231	041110	4	450757, 04409231	040531-1	0	01322
1	153052	041110	4	450758, 04409241	041110	4	450758, 04409241	040532-1	0	01323
1	153053	041110	4	450759, 04409251	041110	4	450759, 04409251	040533-1	0	01324
1	153054	041110	4	450760, 04409261	041110	4	450760, 04409261	040534-1	0	01325
1	153055	041110	4	450761, 04409271	041110	4	450761, 04409271	040535-1	0	01326
1	153056	041110	4	450762, 04409281	041110	4	450762, 04409281	040536-1	0	01327
1	153057	041110	4	450763, 04409291	041110	4	450763, 04409291	040537-1	0	01328
1	153058	041110	4	450764, 04409301	041110	4	450764, 04409301	040538-1	0	01329
1	153059	041110	4	450765, 04409311	041110	4	450765, 04409311	040539-1	0	01330
1	153060	041110	4	450766, 04409321	041110	4	450766, 04409321	040540-1	0	01331
1	153061	041110	4	450767, 04409331	041110	4	450767, 04409331	040541-1	0	01332
1	153062	041110	4	450768, 04409341	041110	4	450768, 04409341	040542-1	0	01333
1	153063	041110	4	450769, 04409351	041110	4	450769, 04409351	040543-1	0	01334
1	153064	041110	4	450770, 04409361	041110	4	450770, 04409361	040544-1	0	01335
1	153065	041110	4	450771, 04409371	041110	4	450771, 04409371	040545-1	0	01336
1	153066	041110	4	450772, 04409381	041110	4	450772, 04409381	040546-1	0	01337
1	153067	041110	4	450773, 04409391	041110	4	450773, 04409391	040547-1	0	01338
1	153068	041110	4	450774, 04409401	041110	4	450774, 04409401	040548-1	0	01339
1	153069	041110	4	450775, 04409411	041110	4	450775, 04409411	040549-1	0	01340
1	153070	041110	4	450776, 04409421	041110	4	450776, 04409421	040550-1	0	01341
1	153071	041110	4	450777, 04409431	041110	4	450777, 04409431	040551-1	0	01342
1	153072	041110	4	450778, 04409441	041110	4	450778, 04409441	040552-1	0	01343
1	153073	04								

1	130431	07	7	H	04300	00000000	00000000	00000000	26790000	0	000001
1	130432	71	00304	200	00000	00000000	70	00000	200	1500	15.662207377154407
1	130433	07	7	H	04300	00000000	00000000	00000000	26710000	0	000002
1	130434	71	00304	200	00000	00000000	70	00000	200	1500	15.662207377154407
1	130435	07	7	H	04300	00000000	00000000	00000000	26710000	0	000003
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1	130440	71	00304	200	00000	00000000	70	00000	200	1500	15.662207377154407
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1	130443	07	7	H	04300	00000000	00000000	00000000	26710000	0	000007
1	130444	71	00304	200	00000	00000000	70	00000	200	1500	15.662207377154407
1	130445	07	7	H	04300	00000000	00000000	00000000	26710000	0	000008
1	130446	71	00304	200	00000	00000000	70	00000	200	1500	15.662207377154407
1	130447	07	7	H	04300	00000000	00000000	00000000	26710000	0	000009
1	130448	71	00304	200	00000	00000000	70	00000	200	1500	15.662207377154407
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1	130450	71	00304	200	00000	00000000	70	00000	200	1500	15.662207377154407
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1	130456	71	00304	200	00000	00000000	70	00000	200	1500	15.662207377154407
1	130457	07	7	H	04300	00000000	00000000	00000000	26710000	0	000014
1	130458	71	00304	200	00000	00000000	70	00000	200	1500	15.662207377154407
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1	130464	71	0030								

[illegible]

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[illegible]

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VITA

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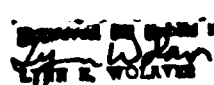
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(19)

This investigation derived a simple model to determine the change in mean motion over time when the actual values are unknown. A method was developed to include effects of solar flux by calculating an average value of \dot{n} over about 30 days. The model requires a knowledge of the mean motion for about 30 days before the time of interest to calculate this average.

The analysis was done using BMDP on a CDC Cyber 6000 Computer using element set data from actual satellites.

This model does not attempt absolute accuracy, but is intended to be a method to quickly approximate a new mean motion when real values are not available. A limitation of this model is the amount of historical data and analyst judgement which are required.

END

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